THREE HINGED ARCHES - 2

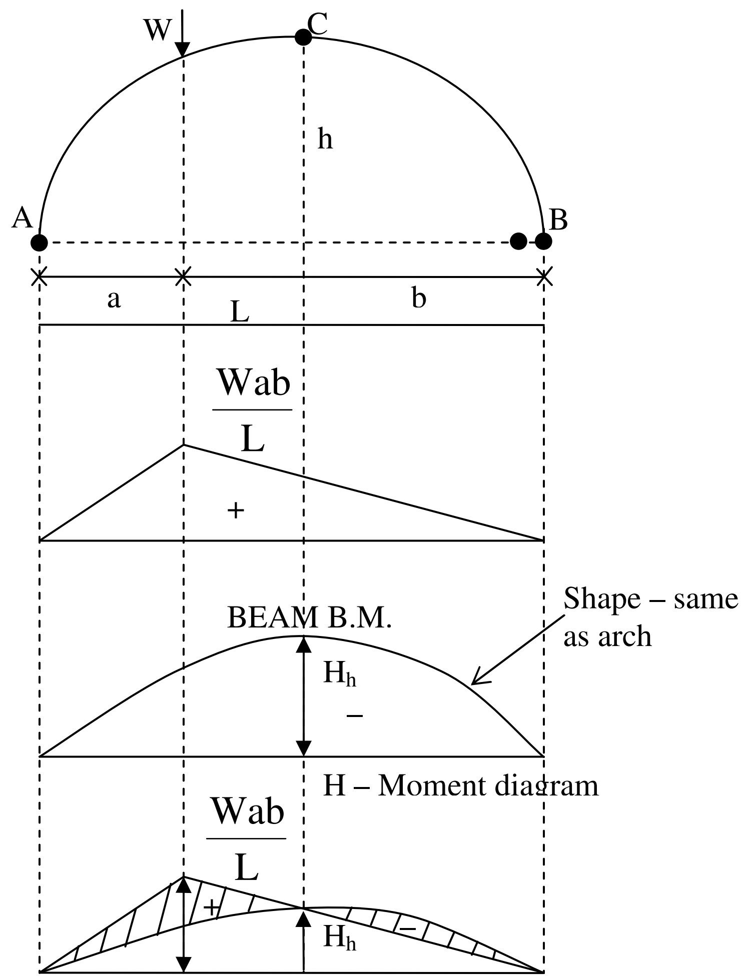
By:

Dr. N Suresh,
NIE, Mysore

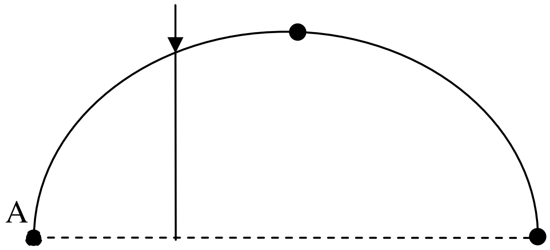
e-Notes for the lecture on VTU EDUSAT Programme

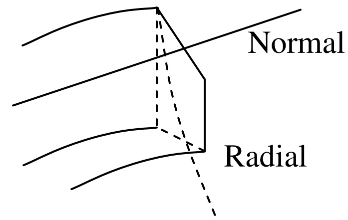
Bending moment diagram for a 3-hinged arch

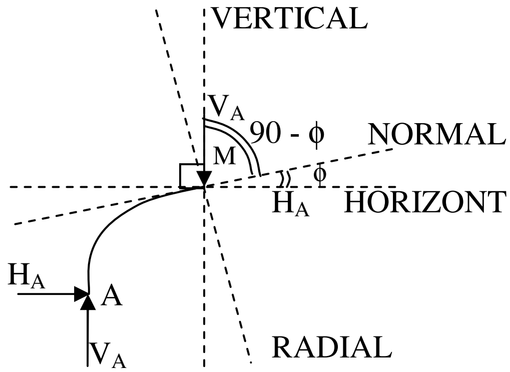
We know that for an arch, bending moment at any point is equal to beam BM-Hy (Refer comparison between arch and beam). Hy is called H-Moment. It varies with respect to Y. Therefore the shape of BM due to Hy should be the shape of the arch. Therefore to draw the BMD for an arch, draw the BMD for the beam over that superimpose the H-moment diagram as shown in fig.

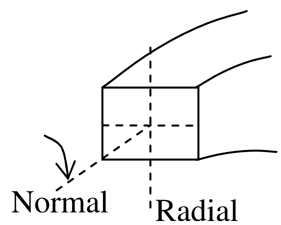


Normal thrust and radial shear in an arch









Total force acting along the normal is called normal thrust and total force acting along the radial direction is called radial shear. For the case shown in fig normal thrust

$= + H_A \text{ Cos } \phi + V_A \text{ Cos } (90 - \phi)$

$= H_A \text{ Cos } \phi + V_A \text{ Sin } \phi$

(Treat the force as +ve if it is acting towards the arch and -ve if it is away from the arch).

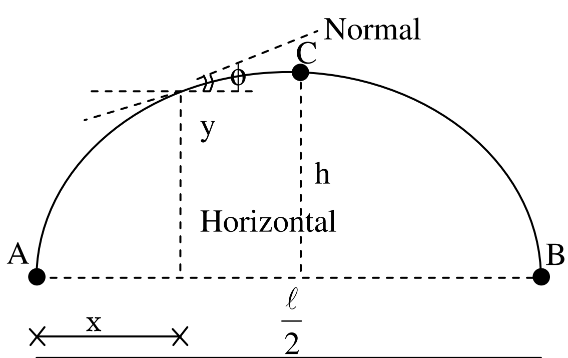
Radial shear $= + H_A \text{ Sin } \phi - V_A \text{ Sin } (90 - \phi)$

$= H_A \text{ Sin } \phi + V_A \text{ Cos } \phi$

(Treat force up the radial direction +ve and down the radial direction as -ve).

Note: 1) To determine normal trust and tangential shear at any point cut the arch into 2 parts. Consider any 1 part. Determine net horizontal and vertical force on to the section. Using these forces calculate normal thrust and tangential shear.

2. Parabolic arch: If the shape of the arch is parabolic then it is called parabolic arch.



If A is the origin then the equation of the parabola is given by y = cx [L – x] where C is a constant.

We have at $X = \frac{L}{2}$ $y = h$

$$h = C\,\frac{L}{2}\left[L - \frac{L}{2}\right] = C.\frac{L}{2}.\frac{L}{2}$$

$$C = \frac{4H}{L^2}$$

Equation of parabola is

$$y = \frac{4hx}{L^2}(L - x)$$

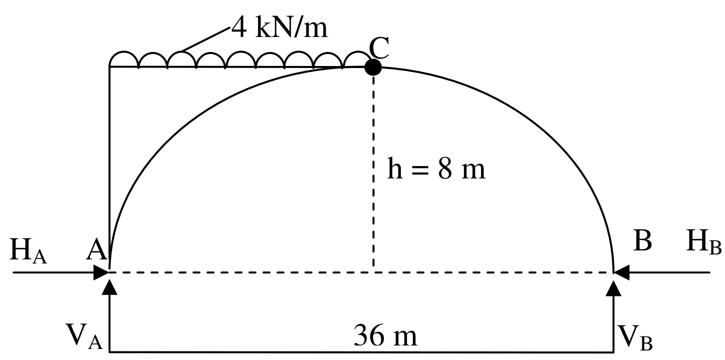
ϕ is given by the following equation.

$$y = \frac{4h}{L^2}(Lx - x^2)$$

$$\tan\phi = \frac{dy}{dx} = \frac{4h}{L^2}(L - 2x)$$

$$\tan\phi = \frac{4h}{L^2}(L - 2x)$$

1. A UDL of 4kN/m covers left half span of 3-hinged parabolic arch of span 36m and central rise 8m. Determine the horizontal thrust also find (i) BM (ii) Shear force (iii) Normal thrust (iv) Radial shear at the loaded quarter point. Sketch BMD.



$$\sum F_x = 0$$

$$H_A - H_B = 0$$

------ (1)

$$H_A = H_B$$

$$\sum F_y = 0$$

$$V_A + V_B = 4 \text{ x } 18$$

------ (2)

$$V_A + V_B = 72$$

$$\sum M_A = 0$$

$$-V_B \text{ x } 36 + 4 \text{ x } 18 \text{ x } 9 = 0$$

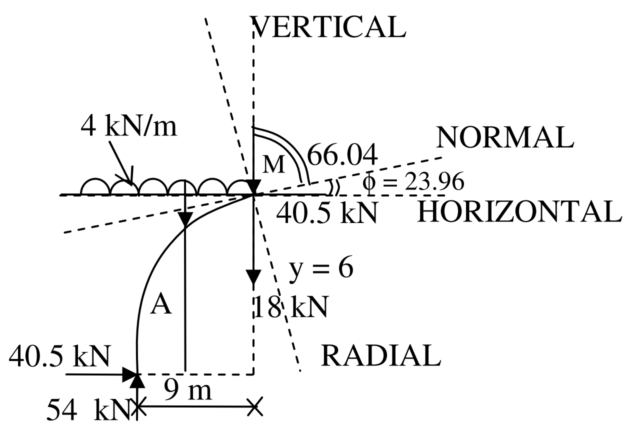
$$V_B = 18\,kN \;\therefore V_A = 54\,kN$$

$$M_c = 0$$

$$+V_B \text{ x } 18 - H_B \text{ x } 8 = 0$$

$$H_B = 40.5\,kN$$

$$H_A = 40.5\,kN$$



BM at M =

- 40.5 x 6 + 54 x 9

- 4 x 9 x 4.5

= 81 kN.m

$$y = \frac{4hx}{L^2}(L - x)$$

$$y = \frac{4 \text{ x } 8 \text{ x } 9}{36^2}(36 - 9)$$

$$y = 6m$$

Shear force at M = + 54 – 4 x 9 = 18 kN (only vertical forces)

$$\tan\phi = \frac{4h}{L^2}(L-2x)$$

$$= \frac{4 \times 8}{36^2}(36 - 2 \times 9)$$

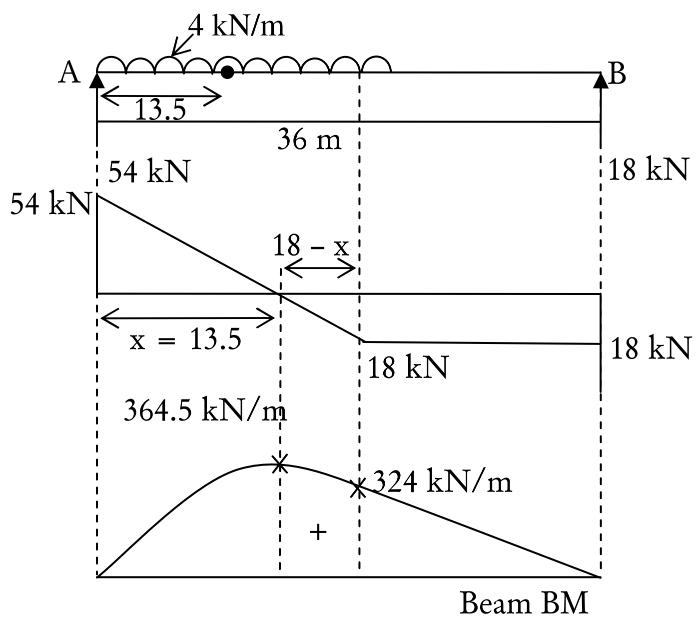
$$\phi = 23^0.96$$

Normal thrust = N = + 40.5 Cos 23.96 + 18 Cos 66.04

= 44.32 kN

S = 40.5 Sin 23.96 – 18 Sin 66.04

S = - 0.0019 ≈ 0



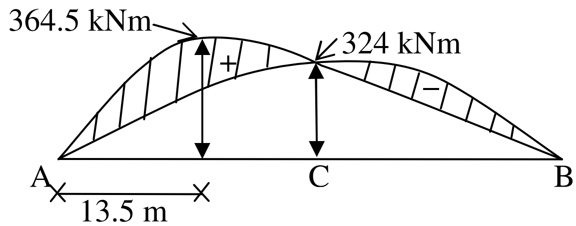
$$\frac{x}{54} = \frac{18-x}{18}$$

$$x = 54 - 3x$$

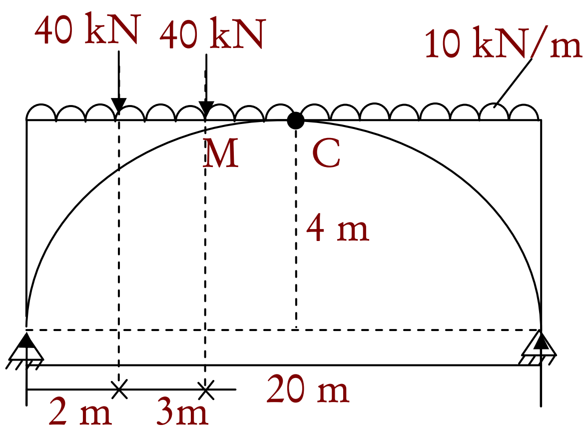
$$x = 13.5m$$

$$54 \times 13.5 - 4 \times 13.5 \times \frac{13.5}{2}$$

$$= 364.5 \text{ kNm}$$



A symmetrical 3-hinged parabolic arch has a span of 20m. It carries UDL of intensity 10 kNm over the entire span and 2 point loads of 40 kN each at 2m and 5m from left support. Compute the reactions. Also find BM, radial shear and normal thrust at a section 4m from left end take central rise as 4m.



$$\sum F_x = 0$$

$$H_A - H_B = 0$$

------ (1)

$$H_A = H_B$$

$$\sum F_y = 0$$

$$V_A + V_B - 40 - 40 - 10 \text{ x } 20 = 0$$

------ (2)

$$V_A + V_B = 280$$

$$\sum M_A = 0$$

$$+40 \text{ x } 2 + 40 \text{ x } 5 + (10 \text{ x } 20)10 - V_B \text{ x } 20 = 0$$

$$V_B = 114 \text{kN}$$

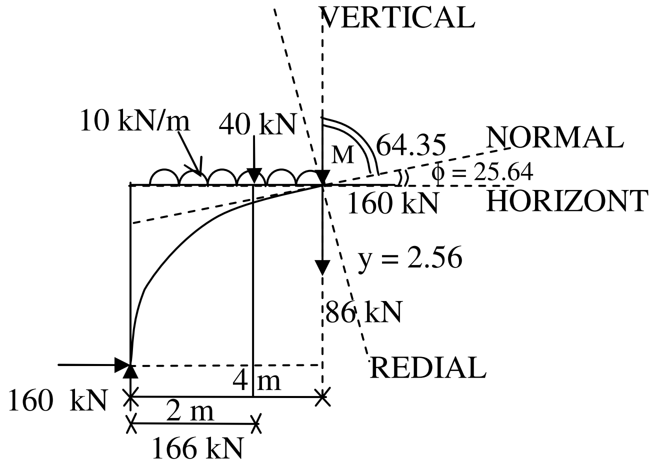
$$V_A = 166 \text{kN}$$

$$M_c = 0$$

$$-(10 \times 10)5 - H_B \times 4 + 114 \times 10 = 0$$

$$H_B = 160\,kN$$

$$H_A = 160\,kN$$



BM at M

= - 160 x 2.56

+ 166 x 4 – 40 x 2

- (10 x 4)2

= + 94.4 kNm

$$y = \frac{4hx}{L^2}(L - x)$$

$$= \frac{4 \times 4 \times 4}{20^2}(20 - 4)$$

$$y = 2.56m$$

$$\tan\phi = \frac{4h}{L^2}(L - 2x)$$

$$= \frac{4 \times 4}{20^2}(20 - 2 \times 4)$$

$$\phi = 25^0.64$$

Normal thrust = N = + 160 Cos 25.64

+ 86 Cos 64.36

= 181.46 kN

S = 160 Sin 25.64

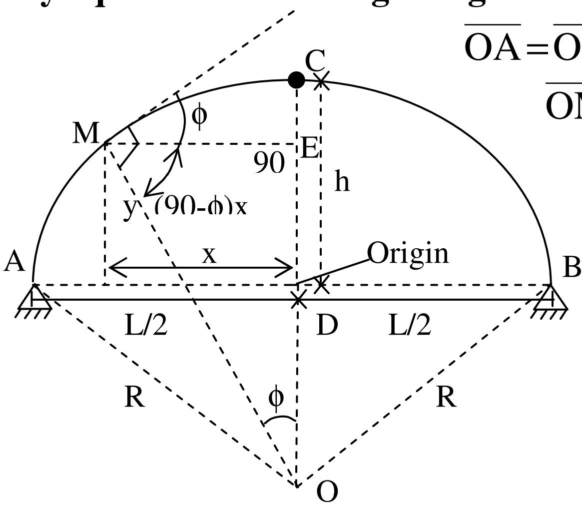
- 86 x Sin 64.36

S = - 8.29 kN

Segmental arch

A segmental arch is a part of circular curve. For such arches $y = \frac{4hx(L-x)}{L^2}$ is not applicable since the equation is applicable only for parabolic arches. Similarly equation for ϕ will be different.

To develop necessary equations for 3-hinged segmental arch



$$\overline{OA} = \overline{OC} = \overline{OB} = R$$

$$\overline{OM} = R$$

Relationship between R, L and h:

From ΔOAD

$$\overline{OA}^2 = \overline{AD}^2 + \overline{OD}^2$$

$$R^2 = \left(\frac{L}{2}\right)^2 + [R-h]^2$$

$$R^2 = \frac{L^2}{4} + R^2 - 2Rh + h^2$$

$$+2Rh = \frac{L^2}{4} + h^2$$

$$R = \frac{L^2}{8h} + \frac{h}{2}$$

$$\sin\phi = \frac{x}{R}$$

$$\cos\phi = \frac{OE}{OM}$$

$$\cos\phi = \frac{R-h+y}{R}$$